

PROTON STRUCTURE IN TRANSVERSE SPACE AND THE EFFECTIVE CROSS SECTION

G. Calucci and D. Treleani

*Università di Trieste; Dipartimento di Fisica Teorica, Strada Costiera 11, Miramare-Grignano,
and INFN, Sezione di Trieste, I-34014 Trieste, Italy.*

Abstract

The observation of double parton collisions by CDF has provided the first direct information on the structure of the proton in transverse space. The actual quantity which has been measured is the ‘effective cross section’ σ_{eff} , which is related to the transverse size of the region where hard interactions are localized. The actual value which has been measured is sizably smaller than naively expected and it is an indication of important correlation effects in the many-body parton distribution of the proton. We discuss the problem pointing out a possible source of correlations in the proton structure, which could have a significant effect on the value of σ_{eff} .

E-mail giorgio@trieste.infn.it

E-mail daniel@trieste.infn.it

I. INTRODUCTION

While parton distributions represent the relevant information on the non perturbative input for most of the large p_t processes at present energy, parton distributions do not exhaust the information on the hadron structure. In fact the information in the parton distributions corresponds to the average number of partons of a given kind and with a given momentum fraction x which is seen when probing the hadron with the resolution Q^2 . In this respect a qualitative change has occurred when it has been shown that the observation of hadronic interactions with multiple parton collisions is experimentally feasible. In a multiparton collision event different pairs of partons interact independently at different points in the transverse plane [1] and the process, as a consequence, depends in a direct way on the actual distribution of the interacting partonic matter in transverse space. The simplest case is the double parton scattering. The non perturbative input to the process is the double parton distribution $D_2(x, x'; \mathbf{b})$ depending on the momentum fractions of the interacting partons and on their relative distance in the transverse plane \mathbf{b} . If all partons are uncorrelated and the dependence on the different degrees of freedom is factorized, one may write $D_2(x, x'; \mathbf{b}) = g_{eff}(x)g_{eff}(x')F(\mathbf{b})$ where $g_{eff}(x)$ is the usual effective parton distribution, namely the gluon plus $4/9$ of the quark and anti-quark parton distributions. When the interacting hadrons are identical $F(\mathbf{b})$ is equal to the overlap in transverse plane of the matter distribution of the two hadrons (normalized to one) as a function of \mathbf{b} , which now represents the impact parameter. The effective cross section σ_{eff} is introduced by the inclusive double scattering process, which is proportional to $1/\sigma_{eff} = \int d^2b F^2(\mathbf{b})$ [1]. The effective cross section is then a well defined property of the hadronic interaction and, at least if the simplest possibility for the hadronic structure is realized, it is both energy and cutoff independent. Analogously the dimensional scale factors, which one may introduce in relation to triple, quadruple etc. partonic collisions, are energy and cutoff independent properties of the interaction.

Initially the search of double parton collisions have been sparse and not very consistent [2]. CDF however has recently claimed the observation of a large number of double parton scatterings [3]. The measured value of the effective cross section, $\sigma_{eff} = 14.5 \pm 1.7_{-2.3}^{+1.7} mb$, is sizably smaller than expected naively and it is an indication of important correlation effects in the hadron structure [4]. Correlations on the other hand are a manifestation of the links between the different constituents of the hadronic bound state and, in fact, it is precisely this sort of connections which one would like to learn as a result of experiments probing the hadron structure.

In the present paper we point out a rather natural possible source of correlations, which is able to give rise to a sizable reduction of the value of the effective cross section, as compared to the most naive expectation. In fact we show that, when linking the transverse size of the gluon and sea distributions to the actual configuration of the valence, the expected value of σ_{eff} is sizably decreased. In the next paragraph we recall the main features of the multiple partonic collision processes in the simplest uncorrelated case, in the following section we correlate the distributions of gluons and sea quarks with the configuration of the valence

and, in the final paragraph, we discuss our results.

II. MULTIPLE PARTON COLLISIONS

The non-perturbative input to the multiple parton collision processes is the many body parton distribution. In most cases the simplest uncorrelated case, with factorized dependence on the actual degrees of freedom, is considered and the probability distribution is expressed as a poissonian. The probability of a configuration with n partons with fractional momenta x_1, \dots, x_n and with transverse coordinates $\mathbf{b}_1, \dots, \mathbf{b}_n$ is then expressed as

$$\Gamma(x_1, \mathbf{b}_1, \dots, x_n, \mathbf{b}_n) = \frac{D(x_1, \mathbf{b}_1) \dots D(x_n, \mathbf{b}_n)}{n!} \exp \left\{ - \int D(x, \mathbf{b}) dx d^2b \right\} \quad (1)$$

where $D(x, \mathbf{b}) = g_{eff}(x)f(\mathbf{b})$ is the average number of partons with momentum fraction x and with transverse coordinate \mathbf{b} . The function $g_{eff}(x)$ is the effective parton distribution while $f(\mathbf{b})$ represents the partonic matter distribution in transverse space. The many body parton distribution is an infrared divergent quantity, one needs therefore to introduce an infrared cut-off. A natural cutoff in this context is the lower value of momentum transfer which allows a scattered parton to be recognized as jet in the final state. Given the many-body parton distribution one can work out all possible multi-parton interactions. If rescatterings are neglected, namely if every parton is allowed to interact with large momentum transfer (namely larger than the infrared cutoff) at most once, one may write a simplest analytic expression for the hard cross section σ_H , corresponding to the cross section counting all inelastic hadronic events with at least one hard partonic interaction [5]:

$$\sigma_H = \int d^2\beta \left[1 - e^{-\sigma_S F(\beta)} \right] = \sum_{n=1}^{\infty} \int d^2\beta \frac{(\sigma_S F(\beta))^n}{n!} e^{-\sigma_S F(\beta)} \quad (2)$$

here β is the impact parameter of the hadronic collision, $F(\beta) = \int d^2b f(\mathbf{b})f(\mathbf{b} - \beta)$ and σ_S is the usual expression for the integrated inclusive cross section to produce jets, namely the convolution of the parton distributions and of the partonic cross section. The unitarized expression of σ_H in Eq.2, which takes into account the possibility of many parton-parton interactions in an overall hadron-hadron collision, is well behaved in the infrared region and it corresponds to a poissonian distribution of multiple parton collisions at a fixed value of the impact parameter of the hadronic interaction. If one works out in Eq.2 the average number of partonic collisions one obtains

$$\langle n \rangle \sigma_H = \int d^2\beta \sigma_S F(\beta) = \sigma_S \quad (3)$$

so that σ_S represents the inclusive cross section normalized with the multiplicity of partonic interactions. The inclusive cross section for a double parton scattering σ_D , normalized in an analogous way with the multiplicity of double parton collisions, $\langle n(n-1)/2 \rangle \sigma_H$, is given by

$$\frac{\langle n(n-1) \rangle}{2} \sigma_H = \frac{1}{2} \int d^2\beta \sigma_S^2 F^2(\beta) = \sigma_D \quad (4)$$

One may then introduce the effective cross section:

$$\sigma_D = \frac{1}{2} \frac{\sigma_S^2}{\sigma_{eff}} \quad (5)$$

which is therefore expressed in terms of the overlap of the partonic matter distribution of the two interacting hadrons as

$$\sigma_{eff} = \frac{1}{\int d^2\beta F^2(\beta)} \quad (6)$$

In fig.1 two different analytic forms for $f(\mathbf{b})$ are compared: a gaussian and the projection of a sphere in transverse space. The radius of the distributions has been fixed in such a way that in both cases the RMS radius has the same value that, following CDF [3], has been set equal to 0.56fm. For the cutoff we have used 5GeV and, in order to be able to reproduce the value of the integrated hard cross section for producing minijets measured by UA1 [6], we have multiplied the partonic cross section, computed in the perturbative QCD parton model, by the appropriate k -factor. Both choices give a similar qualitative result: the effective cross section is constant with energy (and cutoff independent) while the ‘total’ hard cross section σ_H grows rapidly as a function of the c.m. energy. The values obtained for σ_{eff} are however too large, by roughly a factor two, as compared to the value quoted by experiment. One has to add that in the experimental analysis of CDF all events with triple parton collisions have been removed from the sample of inelastic events with double parton scatterings. The resulting value for the effective cross section, $\sigma_{eff}|_{CDF}$, is therefore larger with respect to the quantity discussed here above and usually considered in the literature. The disagreement with the most naive picture is therefore even stronger then apparent when comparing the result of the uncorrelated calculation with the quoted value of σ_{eff} .

In the usually considered picture of multiparton interactions just recalled all correlations are neglected and one may therefore claim that the experimental evidence is precisely that correlations play an important role in the many-body parton distribution of the hadron. In the next paragraph we propose therefore a slight modification to the picture, linking the transverse size of the gluon and sea distributions to the configuration in transverse space of valence quarks.

III. A SIMPLEST MODEL FOR THE PARTONIC MATTER DISTRIBUTION IN THE PROTON

Charged matter is distributed in the proton according to the charge form factor, which is well represented by an exponential expression in coordinate space. The information refers to a large extent to the distribution in space of the valence quarks, which can then be found in various different configurations in transverse space with a given probability distribution. Less is known on the distribution in space of the neutral matter component of the proton structure, namely the gluons. In this respect one may consider two different extreme possibilities:

- the distribution in space of sea quarks and gluons has no relation with the distribution of valence quarks,
- or, rather, its transverse size is linked closely to the actual configuration in space of the valence quarks.

The no correlation hypothesis, which has been ruled out by the measure of σ_{eff} , would imply the first possibility. We try therefore to work out here a simplest model where the distribution of the whole partonic matter in the hadron is driven by the actual configuration of valence quarks.

The parton distribution in Eq.1 can be modified by separating the valence quarks from gluons and sea quarks and by correlating the transverse size of the distributions of sea quarks and gluons with the transverse size of the actual configuration of the valence. If however one simply rescales the distribution function in transverse space $f(\mathbf{b})$ by rescaling the radius while keeping the normalization constant, one is imposing a conservation constraint in the number of gluons. The number is in fact the same in each configuration of the proton, both when it is squeezed to a small transverse size and when it is expanded to a relatively large dimension. The picture therefore is not consistent with the common believe that the energy to the gluon field grows basically because of the growth of the distance between the valence quarks. It seems therefore more reasonable to remove the constraint on normalization and to keep rather fixed the maximum value of $f(\mathbf{b})$ in all configurations. We modify therefore the simplest poissonian expression in Eq.1 and we express the probability distribution Γ as follows:

$$\Gamma(x_1, \mathbf{b}_1, \dots, x_n, \mathbf{b}_n) = \varphi(\mathbf{B}_D, \mathbf{B}) q_v(X_1) q_v(X_2) q_v(X_3) \times \frac{1}{n!} \left[g(x_1) f(b, \mathbf{b}_1) \frac{b^2}{\langle b^2 \rangle} \dots g(x_n) f(b, \mathbf{b}_n) \frac{b^2}{\langle b^2 \rangle} \right] \exp \left\{ - \frac{b^2}{\langle b^2 \rangle} \int g(x) dx \right\} \quad (7)$$

To simplify the notation we have not written explicitly the dependence of Γ on the coordinates of the valence quarks, X and \mathbf{B} . The dependence of Γ on the momentum fraction of the valence quarks X is factorized and $q_v(X)$ is the usual distribution of valence quarks as a function of the momentum fraction X . The function $f(b, \mathbf{b}_i)$ represents the distribution of gluons and sea quarks in transverse space. It is a function of the transverse coordinate of the considered parton, \mathbf{b}_i , and it depends on the scale factor b whose value is given by the actual configuration of the valence in transverse space. The transverse coordinates of the three valence quarks are

$$\begin{aligned} \mathbf{B}_1 &= \frac{1}{2} \mathbf{B}_D + \mathbf{B} \\ \mathbf{B}_2 &= \frac{1}{2} \mathbf{B}_D - \mathbf{B} \\ \mathbf{B}_3 &= -\mathbf{B}_D \end{aligned} \quad (8)$$

The dependence on the transverse coordinates of the valence is given by $\varphi(\mathbf{B}_D, \mathbf{B})$, that is the integral on the longitudinal coordinates Z_D, Z of $\phi(\mathbf{R}_D, \mathbf{R})$, representing the valence structure of the proton in coordinate space. Explicitly we use the exponential form

$$\phi(\mathbf{R}_D, \mathbf{R}) = \frac{\lambda_D^3 \lambda^3}{(8\pi)^2} \exp \{ - (\lambda_D R_D + \lambda R) \} \quad (9)$$

where

$$\begin{aligned} \lambda_D &= \frac{2\sqrt{3}}{\sqrt{\langle r^2 \rangle}} \\ \lambda &= \frac{4}{\sqrt{\langle r^2 \rangle}} \end{aligned} \quad (10)$$

and $\sqrt{\langle r^2 \rangle} = 0.81 \text{fm}$ is the proton charge radius.

In a given configuration of the valence, the average number of gluons and sea quarks is not equal to the overall average number $g(x)$ (with $g(x)$ we indicate here the sum of gluon and sea quark distributions). It is rather equal to $g(x) \frac{b^2}{\langle b^2 \rangle}$; where $\langle b^2 \rangle$ is the average of b^2 with the probability distribution of the valence. To make a definite choice we take $b = B_D$ in such a way that

$$\langle b^2 \rangle = \langle B_D^2 \rangle = \int d\mathbf{R}_D d\mathbf{R} B_D^2 \phi(\mathbf{R}_D, \mathbf{R}) \quad (11)$$

The density of gluons and sea quarks in transverse space is then constant in the middle of the proton in all different configurations of the valence quarks and it is equal to the value assumed in the average configuration. The actual number of gluons and sea quarks is therefore a function of the configuration taken by the valence, compact configurations giving rise to small numbers while more extended configurations giving rise to larger numbers.

Summing over all possible probability configurations of gluons and sea quarks one obtains, from Eq.7, the probability distribution of valence in transverse space $\varphi(\mathbf{B}_D, \mathbf{B})$. The average number of gluons and sea quarks, with given momentum fraction x , is

$$\int d\mathbf{B}_D d\mathbf{B} d\mathbf{b} \varphi(\mathbf{B}_D, \mathbf{B}) g(x) f(B_D, \mathbf{b}) \frac{B_D^2}{\langle B_D^2 \rangle} = g(x) \quad (12)$$

while the average number of partonic collisions, non involving valence $\langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle$, for a given configuration of the valence and for a given value of the impact parameter β , is expressed as

$$\langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle = \int d\mathbf{b} f(B_D, \mathbf{b} - \beta) \frac{B_D^2}{\langle B_D^2 \rangle} f(B'_D, \mathbf{b}) \frac{(B'_D)^2}{\langle B_D^2 \rangle} [\sigma_S(g + q_s, g + q_s)] \quad (13)$$

The average number of interactions of valence quark with gluons and sea is written in an analogous way. If one integrates the average number of partonic collisions $\langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle$ over all configurations of the valence quarks of the two hadrons, with the corresponding weights, and on the hadronic impact parameter β , one obtains the single scattering inclusive cross section σ_S :

$$\int d\mathbf{B}_D d\mathbf{B} d\mathbf{B}'_D d\mathbf{B}' d\beta \varphi(\mathbf{B}_D, \mathbf{B}) \varphi(\mathbf{B}'_D, \mathbf{B}') \langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle = \sigma_S(g + q_s, g + q_s) \quad (14)$$

An analogous expressions for interactions of valence quarks with gluons and sea quarks is readily written.

All average quantities are therefore the well known ones of the perturbative QCD-parton model. The inclusion of the transverse degrees of freedom in the relations above allows one to write down promptly all expressions corresponding to the various multiparton collision processes. For the double parton scattering case one has:

$$\sigma_D = \frac{1}{2} \int d\mathbf{B}_D d\mathbf{B} d\mathbf{B}'_D d\mathbf{B}' d\beta \varphi(\mathbf{B}_D, \mathbf{B}) \varphi(\mathbf{B}'_D, \mathbf{B}') [\langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle]^2 \quad (15)$$

and for the hard cross section

$$\sigma_H = \int d\mathbf{B}_D d\mathbf{B} d\mathbf{B}'_D d\mathbf{B}' d\beta \varphi(\mathbf{B}_D, \mathbf{B}) \varphi(\mathbf{B}'_D, \mathbf{B}') [1 - \exp\{-\langle n(\mathbf{B}_D, \mathbf{B}, \mathbf{B}'_D, \mathbf{B}', \beta) \rangle\}] \quad (16)$$

The effective cross section is obtained, as in the uncorrelated case, from σ_D by using Eq.5.

IV. DISCUSSION

The values of σ_H and of σ_{eff} , derived from the correlated parton distribution described in the previous paragraph, are plotted in fig.2, where two different analytic expressions for $f(\mathbf{b})$ are considered. In fig.2a $f(\mathbf{b})$ is the projection of a sharp-edged sphere in transverse space, while in fig.2b $f(\mathbf{b})$ is a gaussian. In each figure we draw the inclusive cross section σ_S , which is the fast growing short-dashed curve, the hard cross section σ_H , continuous curves, and the effective cross section σ_{eff} , long-dashed curves. Both for σ_H and for σ_{eff} we draw two different curves. The higher curve refers to the case where the gluon and sea distributions are kept fixed and equal to the average configuration, irrespectively of the configuration of the valence, while the lower curve refers to the case where the configuration of gluons and sea are correlated in radius with the configuration of the valence, as described in the previous paragraph. The lower threshold for the transverse momentum of the produced jets has been put equal to 5GeV. In the correlated case, when considering a sphere for $f(\mathbf{b})$, we choose for the radius of the sphere the value of B_D . The RMS of the radius of the sphere, averaged with the probability distribution of the valence, is then $\sqrt{\langle B_D^2 \rangle} = 0.66\text{fm}$, namely it is equal to $\sqrt{2/3 \langle r^2 \rangle}$, with $\sqrt{\langle r^2 \rangle} = 0.81\text{fm}$, the RMS proton charge radius. In the gaussian case we fix the size of the distribution by requiring the same RMS value for the radius of the distribution as in the case of the sphere. As one may see in fig.2, while the value of σ_{eff} is too large when the distribution of gluons and sea quarks is kept fixed, the value of σ_{eff} is sizably reduced when it is correlated with the distribution of the valence quarks.

With our choice the value of the effective cross section turns out of the right size. Obviously the result could have been significantly different with a different, but still plausible choice of the (correlated) radius of the gluon and sea distributions. We do not claim therefore that the simplest mechanism discussed here is the only solution to the problem posed by the smallness of the observed value of σ_{eff} . Rather we point out that the source of correlations discussed here on one side is a minimal modification to the uncorrelated many-body

parton distribution and, on the other, it looks as a natural possibility, which could explain a substantial amount of the difference between the observed value of σ_{eff} and the result of the uncorrelated calculation.

As shown in fig.2, our simplest model, while reducing sizeably the expectation for σ_{eff} , does not modify dramatically the expectation for σ_H . It is therefore worth while making predictions for other possible observables, in order to have an independent indication on the actual model. We have then estimated the triple parton scattering cross section. The triple scattering cross section, being proportional to σ_S^3 , introduces a new dimensional quantity other than the effective cross section σ_{eff} . In the uncorrelated case the new dimensional quantity has a given value, proportional to σ_{eff}^2 and the proportionality factor depends on the actual form of $f(\mathbf{b})$. Also in the correlated case one may however write

$$\sigma_T = \frac{1}{3!} \frac{\sigma_S^3}{\tau \sigma_{eff}^2} \quad (17)$$

The observation of the triple scattering parton process allows therefore one to measure the dimensionless quantity τ , which, as σ_{eff} is a well defined quantity, related to the geometrical properties of the interacting hadrons. The actual expectations for the τ -factor according to the model discussed here are shown in the table.

V. CONCLUSIONS

The observation of multiple parton collisions allows the measure of a whole set of quantities which characterize the interaction and that are directly connected to the geometrical properties of the hadron structure. The first indication in this direction is the measure of the effective cross section of double parton collisions performed by CDF. The measured value of the effective cross section rules out the simplest uncorrelated picture of the many body parton distribution of the hadron. In this note we have shown that correlating the transverse dimension of the gluon and sea quark distributions to the transverse dimension of the valence one can obtain for the effective cross section a value much closer to the experimental indication. In order to have the possibility to test the model, at least to some extent, we have then worked out our prediction for the factor τ , characterizing the triple parton scattering cross section.

The simplest expectation for σ_{eff} and, more in general, for the scale factors of the different multiple parton collision processes, is that they are cutoff and energy independent. The situation however is changed when more elaborate structures are considered. Also in the simplest model discussed here in fact σ_{eff} shows a slight energy and cutoff dependence. The origin is the following: In our case partons are organized in two different structures in transverse space, on one side one has valence quarks and on the other gluons and sea quarks. When changing the energy or the lower cutoff in p_t one is varying the relative number of interacting sea quarks and gluons with respect to the valence quarks. This variation is then

reflected in the relative weight of the rate of collisions of partons which have a different distributions in transverse space and, as a consequence, σ_{eff} is slightly modified.

Acknowledgements

This work was partially supported by the Italian Ministry of University and of Scientific and Technological Research by means of the Fondi per la Ricerca scientifica - Università di Trieste.

TABLES

TABLE I. Factor τ for the triple parton scattering process

Sphere (no corr.)	Gauss (no corr.)	Sphere (with corr.)	Gauss (with corr.)
0.78	0.74	0.5	0.46

FIGURES

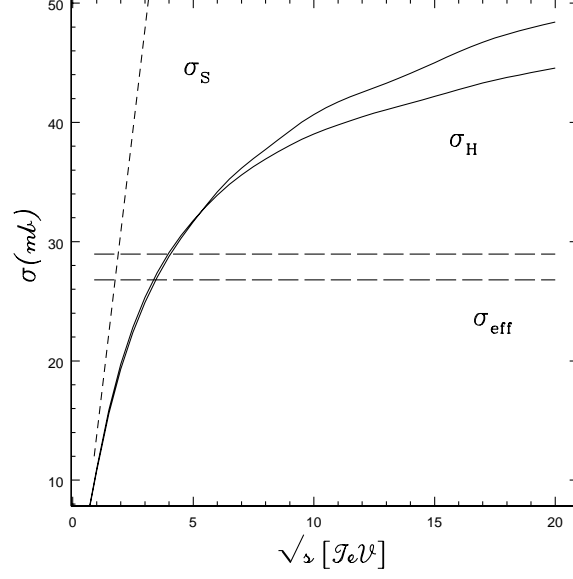


FIG. 1. σ_S , σ_H and σ_{eff} in the simplest uncorrelated case. The two curves for σ_H and σ_{eff} refer to the two different choices made for $f(\mathbf{b})$, a gaussian (higher curve) and a sphere (lower curve).

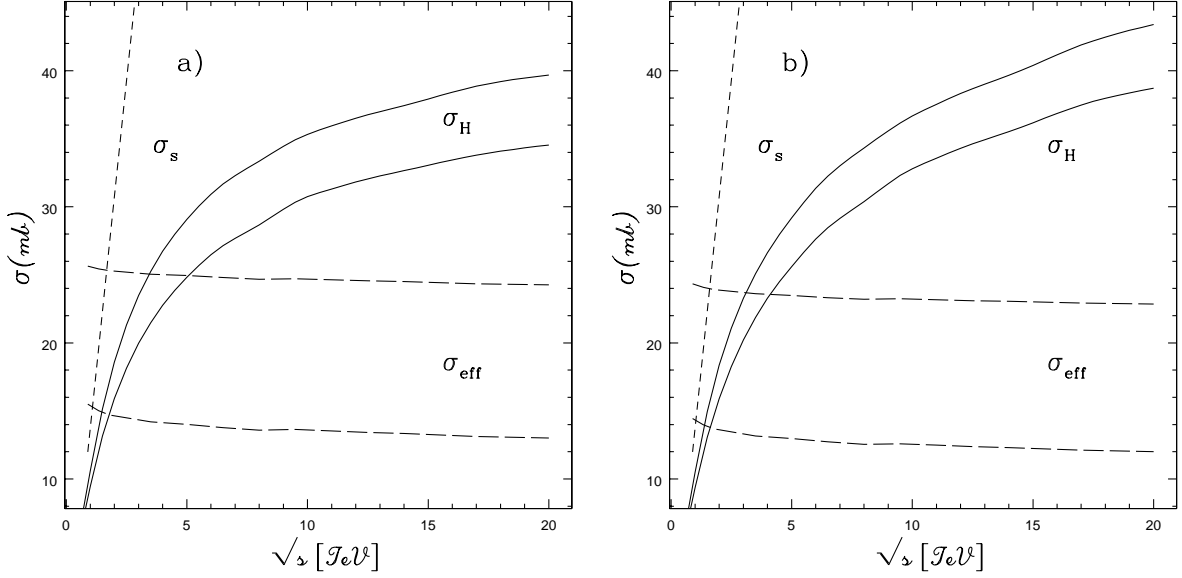


FIG. 2. σ_S , σ_H and σ_{eff} when taking for $f(\mathbf{r})$ a sphere a) or a gaussian b). The lower curves for σ_H and σ_{eff} refer to the correlated many-body parton distribution, the higher curves to the uncorrelated ones.

REFERENCES

- [1] P.V. Landshoff and J.C. Polkinghorne, *Phys. Rev.* **D18** 3344 (1978); Fujio Takagi *Phys. Rev. Lett.* **43**, 1296 (1979); C. Goebel, F. Halzen and D.M. Scott, *Phys. Rev.* **D22**, 2789 (1980); N. Paver and D. Treleani, *Nuovo Cimento* **A70**, 215 (1982); B. Humpert, *Phys. Lett.* **B131**, 461 (1983); M. Mekhfi, *Phys. Rev.* **D32**, 2371 (1985), *ibid.* **D32**, 2380 (1985); B. Humpert and R. Odorico, *Phys. Lett.* **154B**, 211 (1985); T. Sjostrand and M. Van Zijl, *Phys. Rev.* **D36**, 2019 (1987); F. Halzen, P. Hoyer and W.J. Stirling *Phys. Lett.* **188B**, 375 (1987); R.M. Godbole, Sourendu Gupta and J. Lindfors, *Z. Phys.* **C47** 69 (1990).
- [2] T. Akesson et al. (AFS Collaboration), *Z. Phys.* **C34** 163 (1987); J. Alitti et al. (UA2 Collaboration), *Phys. Lett* **B268** 145 (1991); F. Abe et al. (CDF Collaboration), *Phys. Rev.* **D47** 4857 (1993).
- [3] F. Abe et al. (CDF Collaboration), *Phys. Rev. Lett.* **79** 584 (1997); *Phys. Rev.* **D56** 3811 (1997).
- [4] G. Calucci and D. Treleani, *Phys. Rev.* **D57** 503 (1998).
- [5] L. Durand and Pi Hong, *Phys. Rev. Lett.* **58** 303 (1987); A. Capella, J. Tran Thanh Van and J. Kwiecinski *Phys. Rev. Lett.* **58** 2015 (1987); Ll. Ametller and D. Treleani, *Int. J. Mod. Phys.* **A3**, 521 (1988); N. Brown, *Mod. Phys. Lett.* **A4** 2447 (1989).
- [6] C. Albajar et al. *Nucl. Phys.* **B309**, 405 (1988).